Chaos-Based Symmetric Key Cryptosystems

Christopher A. Wood

July 19, 2011

Outline

History of symmetric key ciphers

- Data Encryption Standard (DES) selected as an official FIPS standard in 1976
  - Brute force attacks require a minimum of 2^{56} steps due to key sizes of 56 bits - now feasible thanks to Moore's Law
  - Cryptanalysis revealed weaknesses in the design that could reduce the time complexity of a successful attack to $2^{29.2}$
  - Several replacement ciphers were proposed, including 2DES, 3DES, Blowfish, RC5, and IDEA

History of symmetric key ciphers

- Data Encryption Standard (DES) selected as an official FIPS standard in 1976
  - Brute force attacks require a minimum of 2^{56} steps due to key sizes of 56 bits - now feasible thanks to Moore's Law
  - Cryptanalysis revealed weaknesses in the design that could reduce the time complexity of a successful attack to $2^{29.2}$
  - Several replacement ciphers were proposed, including 2DES, 3DES, Blowfish, RC5, and IDEA
- Advanced Encryption Standard (AES) competition held to replace the aging DES cipher
  - 5 year competition with 15 different symmetric key design proposals, including Rijndael, Twofish, RC6, and Serpent
  - Rijndael selected as the winner - deemed to have the best balance between speed, security, and simplicity

AES (Rijndael)

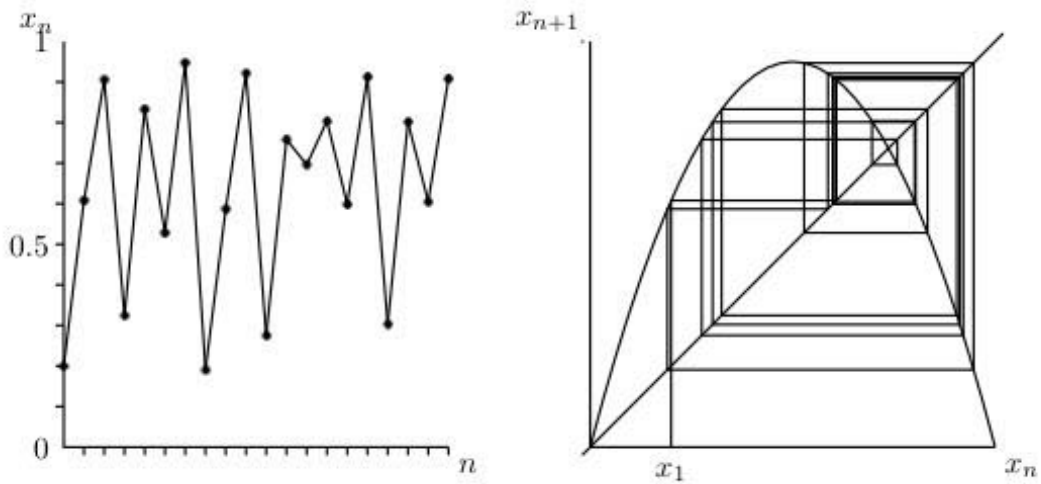
- Round-based symmetric key cipher that operates on blocks of 128 bits
- Key sizes of 128, 192, and 256 bits for varying security
  - Makes brute force attacks infeasible given current computing limitations
- Operates on elements in $GF(2^8)$ defined by the irreducible polynomial $x^8 + x^4 + x^3 + x + 1$
- Four internal operations: 1) AddRoundKey, 2) SubBytes, 3) ShiftRows, and 4) MixColumns

Chaos dynamics

Chaotic systems are characterized by the following properties:

1. Sensitivity to initial conditions
2. Dense period trajectory orbits
3. Topologically mixing

Logistic map



$$x_{n+1} = rx_n(1 - (x_n))$$

- Discrete time-based recurrence relation derived from the differential logistic equation
- Operates in a continuous domain (usually $\mathbb{R}$)
- r and x_0 are the initial conditions of the system

Mapping chaos theory to cryptography

Chaos Theory	**Cryptography**
Mixing	Diffusion
Iterations	Rounds
Initial conditions	Keys

Biggest difference: chaos only truly has meaning in continuous domains, whereas symmetric key cryptosystems operate in finite domains

Chaos-based cipher design

- Chaotic maps are non-linear transformations
  - Can be used to replace other non-linear transformation steps in the cipher (e.g. S-box substitution)
- Chaotic maps are topologically mixing
  - Add diffusion to mixing transformations in the cipher (e.g. MixColumns in AES)
- Chaotic maps are sensitive to initial conditions
  - Can be exploited to provide pseudorandomness to cipher operations (e.g. key generation, non-linear confusion routines)

Limitations

- No formal definition for discrete chaos in finite domains
- Chaotic behavior is approximated using one or more non-linear maps in space-discretized (finite) domains
- Proper security analysis must include Hamming and Euclidean distance measures for chaotic maps

Cipher evaluation

- Security is measured from a theoretical and practical perspective
  - Theoretical - ciphers possess "randomness increasing" and "computationally unpredictable" characteristics
  - Practical - ciphers are resistant to known attacks
- An analysis of the entropy of a cipher is a good indication of its pseudorandom properties
- Resistance to differential and linear cryptanalysis attacks is necessary

Cipher evaluation

- Security is measured from a theoretical and practical perspective
  - Theoretical - ciphers possess "randomness increasing" and "computationally unpredictable" characteristics
  - Practical - ciphers are resistant to known attacks
- An analysis of the entropy of a cipher is a good indication of its pseudorandom properties
- Resistance to differential and linear cryptanalysis attacks is necessary
  - Ciphers should also be strong against trajectory-based, loss of information, and memory attacks
  - Statistical tests can be conducted to determine a PDF for elements of a trajectory

Case studies

- Simple and Advanced Ciphers
- Rabbit Cipher
- Chaotic Feistel Cipher (not included in this talk)

Simple Cipher

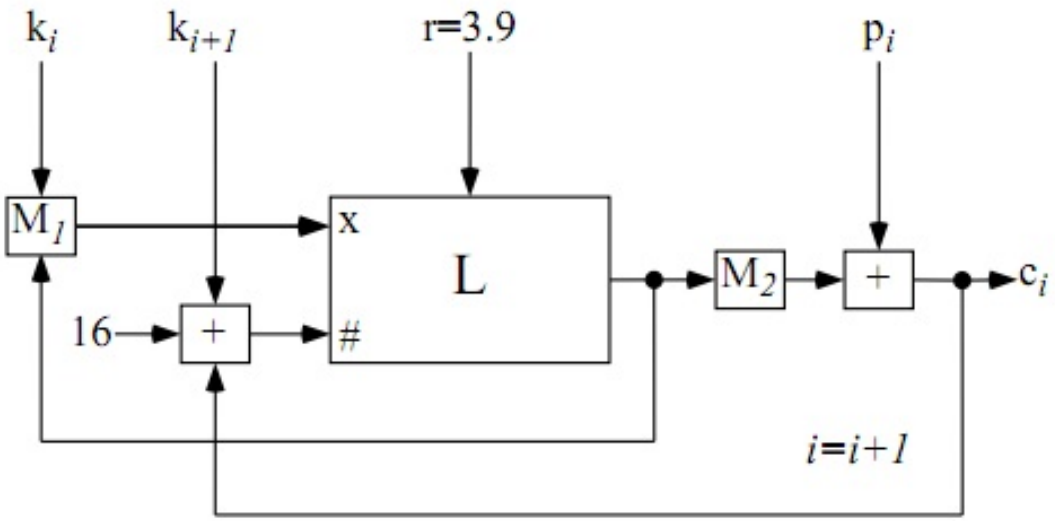
- Design concepts
  - Based on the logistic map for its non-linear transformation
  - Uses the secret key to generate initial conditions for the map
  - Translates elements from the set of keys (2^8) to elements in $(0, 1]$ for the map

Simple Cipher

- Design concepts
  - Based on the logistic map for its non-linear transformation
  - Uses the secret key to generate initial conditions for the map
  - Translates elements from the set of keys (2^8) to elements in $(0, 1]$ for the map
- Design flaws
  - Uses "real" numbers in the logistic map - expensive FLOPS lead to poor performance
  - Level of chaotic behavior tied to the amount of precision in which real numbers are stored
  - Periodic behavior induced by system reliance on keys alone - leads to information leakage

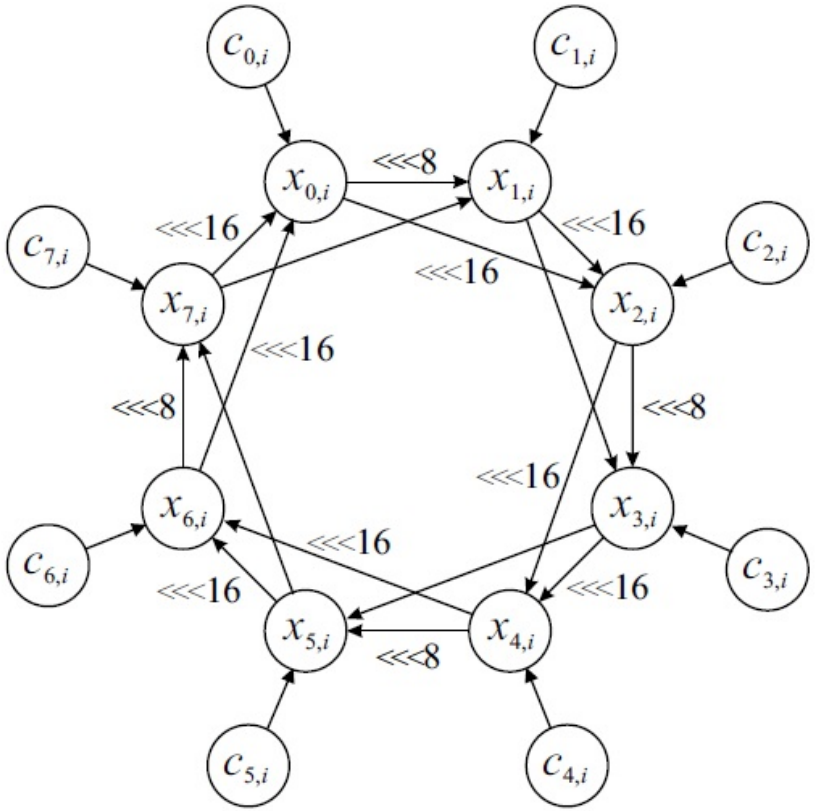
Advanced Cipher

- Enhancement to the Simple Cipher
- Provides a feedback element for increased pseudorandomness and diminished periodic behavior
- Was shown that a single bit change in the encryption key effected approximately 49.6% of the ciphertext bits



Rabbit

- Approximates chaos with a system of eight coupled non-linear maps (combined into the next-state function)



Rabbit

- Works with a phase space of 2^{32} bits
- Does not rely on real number approximations - only uses integers
- Hamming distance analysis revealed high levels of entropy for the chaotic map
- Periodic behavior and algebraic analysis efforts were also done
  - Approached the analysis from a cryptographic perspective and dynamical systems perspective

Conclusion

Chaos-based symmetric key ciphers struggle for success:

- Lack of definition for discrete chaos in finite domains
- Inefficiencies of current implementations (typically related to FLOPS and real number representations)
- Thorough security analysis is difficult and often times indicates that chaos-based ciphers are not comparable in security to standardized ciphers